Saskatchewan
Learning

Grade 4 Mathematics Curriculum

Recycled Paper

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Saskatchewan Learning
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Table of Contents

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- other field personnel.

This curriculum in based on the Western and Northern Canadian Protocol (WNCP) Common Curriculum Framework (CCF) (2006) for Kindergarten to Grade 9 Mathematics.

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Grade Four Mathematics

Purpose

The Grade 4 mathematics curriculum defines the outcomes to be attained by grade four students during the 210 minutes of instruction and learning time allocated per week for the entire school year. The curriculum is designed to support teachers in providing students with the learning opportunities to develop appropriate mathematics knowledge, understandings, and abilities within a learning environment that supports the students' development of positive attitudes and beliefs towards mathematics. Indicators are included for each of the outcomes in order to clarify for the teacher the breadth and depth of learning intended by the outcome. These indicators are a representative list of the kinds of things a student needs to know and/or be able to do in order to achieve the learnings intended by the outcome.

Outcomes are statements identifying what students are expected to know, understand, and be able to do by the end of a particular grade level.

This curriculum also provides information about how the outcomes of the Grade 4 mathematics curriculum connect to the K-12 Goals for mathematics, the Cross-curricular Competencies to be addressed in all areas of study, and the Broad Areas of Learning that summarize the Goals of Education for Saskatchewan students.

Indicators are a representative list of things students could be asked to know or do in order to show their attainment of the outcome. They are intended to clarify the breadth and depth of the outcome.

Appendix A provides explanations of some of the mathematical terminology you will find in this curriculum. Appendix B: Three Grades at a Glance has been included to help teachers see how the Grade 4 outcomes are related to the Grade 3 and 5 outcomes.

This curriculum also provides an introduction to pedagogical understandings necessary for the effective teaching of mathematics. Additional support resources that explore and demonstrate these pedagogical understandings will be provided online.

This curriculum has been designed to address current research in mathematics education as well as to address the learning needs of young children. The Mathematics Grade 4 Curriculum outcomes have been influenced by changes to the outcomes in K-3 mathematics, and will also impact the content of the Grades 5 – 12 mathematics outcomes. Changes throughout all of the grades have been made for a number of reasons including:

- decreasing content in each grade to allow for more depth of understanding
- rearranging concepts to allow for greater depth of learning in one year and to align related mathematical concepts

- increasing the focus on numeracy (understanding numbers) beginning in Kindergarten
- introducing algebraic thinking earlier.

Aim and Goals of K-12 Mathematics

When we ask good questions in math class, we invite our students to think, to understand, and to share a mathematical journey with their classmates and teachers alike. Students are no longer passive receivers of information when asked questions that challenge their understandings and convictions about mathematics. (Sullivan, 2002, p. 1)

The aim of the mathematics program is to prepare individuals who value mathematics and appreciate its role in society. The K-12 mathematics curricula are designed to prepare students to cope confidently and competently with everyday situations that demand the use of mathematical concepts including interpreting quantitative information, estimating, performing calculations mentally, measuring, understanding spatial relationships, and problem solving. The mathematics program is intended to stimulate the spirit of inquiry within the context of mathematical thinking and reasoning.

The four goals for K-12 mathematics are broad statements that identify the knowledge, understandings, skills, and attitudes in mathematics that students are expected to develop and demonstrate by the end of grade twelve. Within each grade level, outcomes are directly related to the development of one or more of these goals. The goals for K-12 mathematics are:

Logical Thinking: *Develop and be able to apply mathematical reasoning processes, skills, and strategies to new situations and problems*.

This goal encompasses the processes and strategies that are foundational to understanding mathematics as a discipline. These processes and strategies include:

- inductive and deductive thinking
- abstracting and generalizing
- exploring, identifying, and describing patterns
- verifying and proofing
- exploring, identifying, and describing relationships
- modeling and representing (concretely, visually, physically, and symbolically)
- hypothesizing and asking “what if” (mathematical play).

Number Sense: *Develop an understanding of the meaning of, relationships between, properties of, roles of, and representations (including symbolic) of numbers and apply this understanding to new situations and problems.*

Key to developing number sense is students having ongoing experience with:

- decomposing and composing of numbers
- relating different operations to each other
- modeling and representing numbers and operations (concretely, visually, physically, and symbolically)
- understanding the origins and need for different types of numbers
- recognizing operations on different number types as being the same operations
- understanding equality and inequality
- recognizing the variety of roles for numbers
- understanding algebraic representations and manipulations in terms of extending numbers
- looking for patterns and ways to describe those patterns numerically and algebraically.

Spatial Sense: *Develop an understanding of 2-D shapes and 3-D objects and the relationships between geometrical shapes and objects, and numbers and apply this understanding to new situations and problems.*

Development of a strong spatial sense requires students to experience:
- construction and deconstruction of 2-D shapes and 3-D objects
- investigations into relationships between 2-D shapes and 3-D objects
- explorations of how numbers (and algebra) can be used to describe 2-D shapes and 3-D objects
- exploration of the movement of 2-D shapes and 3-D objects
- exploration of the dimensions of 2-D shapes and 3-D objects
- exploration of different forms of measurement and their meaning.

Math makes sense! This is the most fundamental idea that an elementary teacher of mathematics needs to believe and act on. It is through the teacher's actions that every child in his or her own way can come to believe this simple truth and, more importantly, believe that he or she is capable of making sense of mathematics. (Van de Walle & Lovin, 2006, p. ix)

Mathematical Attitude: *Develop a positive attitude towards their ability to understand mathematics and to use it to solve problems.*

Mathematical ability and confidence is built through playing with numbers and related concepts in a supportive environment. Students can persevere when challenged if provided with opportunities to learn mathematics within an environment that:
- supports risk taking (mathematically and personally)
- honours students' ideas
- provides engaging and responsive learning experiences.

Students who have a positive attitude towards mathematics demonstrate:
- confidence in their mathematical insights and abilities
- enjoyment, curiousity, and perseverance when encountering new problems

- appreciation of the structure and value of mathematics.

Although there are many "real-world" applications of the mathematics within the K-12 mathematics program, the curriculum content first and foremost serves as the vehicle through which students can achieve the four goals of K-12 mathematics in Saskatchewan. Attainment of these four goals will result in students with mathematical confidence and the tools necessary to succeed in any future mathematical endeavours that students pursue and any mathematical problems encountered.

Connections to Broad Areas of Learning

There are three Broad Areas of Learning that reflect Saskatchewan's Goals of Education. K-12 mathematics contributes to the Goals of Education through helping students achieve knowledge, skills, and attitudes related to these Broad Areas of Learning.

Building a Disposition for Learning

Students who are provided with a variety of ways to seek, use, evaluate, and create information are able to approach learning with curiosity, flexibility, and perseverance.

Students who are engaged in constructing and applying mathematical knowledge naturally build a positive disposition towards learning. Throughout their study of mathematics, students should be learning the skills (including reasoning strategies) and developing the attitudes that will enable the successful use of mathematics in daily life. Moreover, students should be developing understandings of mathematics that will enable the learning of new mathematical concepts and applications that may be encountered within both career and personal interest choices. Students who successfully complete their study of K-12 mathematics should feel confident about their mathematical abilities and have developed the knowledge, understandings, and abilities necessary to make future use and/or studies of mathematics meaningful and attainable. In order for mathematics to contribute to this Broad Area of Learning, students must actively learn the mathematical content in the outcomes through using and developing mathematical reasoning, number sense, spatial sense, and mathematical attitude (the four goals of K-12 mathematics). It is crucial that the *students uncover* the mathematics outlined in the curriculum rather than the *teacher covering* it.

Mathematics enables individuals to understand and explore their world, and communicate and participate in a variety of roles and settings in their home, school, and community.

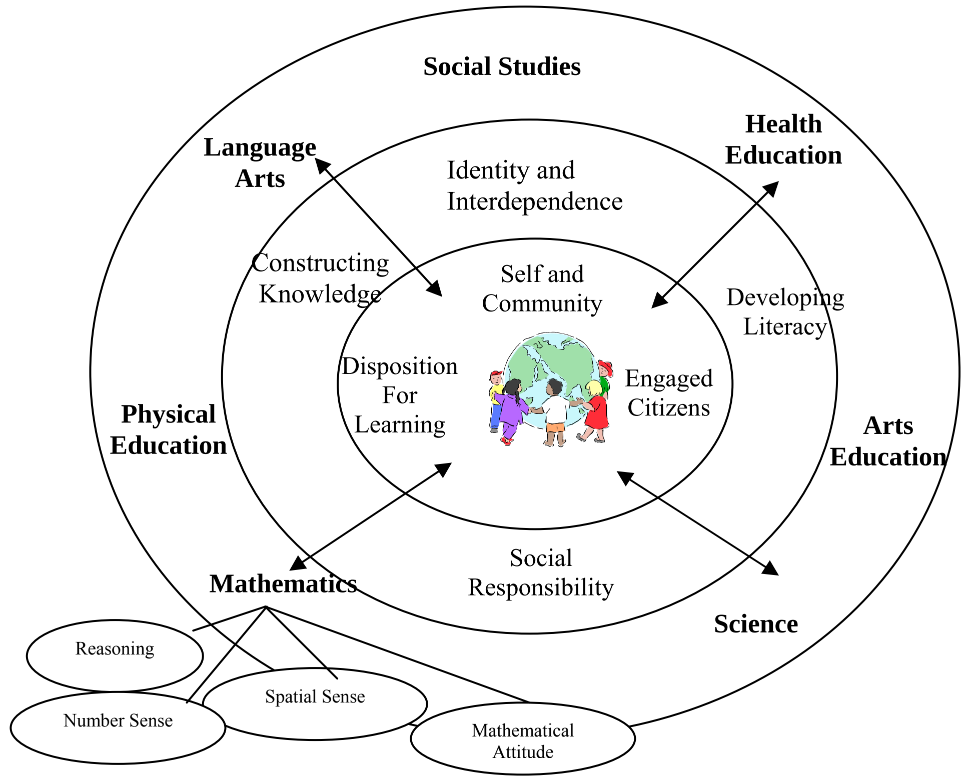
Building a Sense of Self and Community

To learn mathematics for deep understanding, students need to not only interact with the mathematical content but with each other as well. Mathematics needs to be taught in a dynamic environment where students work together to share and evaluate strategies and understandings. Students who are involved in a supportive mathematics learning environment that is rich in dialogue are exposed to a wide variety of perspectives and strategies from which to construct a sense of the mathematical content. In such an environment, students also learn and come to value how they as individuals and as members of a group or community can contribute to understanding and social well-being through a sense of accomplishment, confidence, and relevance. When encouraged to present ideas that represent different perspectives and ways of knowing, students in mathematics classrooms develop a deeper understanding of the mathematics. At the same time students also learn to respect and value the contributions of others. Mathematics also provides many opportunities for students to enter into communities beyond the classroom walls by engaging with people in the neighbourhood or around the world. By working towards the common goal of developing a deeper understanding of mathematics and its role in the world, students will necessarily develop their personal and social identity, and learn healthy and positive ways of interacting and working together with others.

Building Engaged Citizens

Mathematics brings a unique perspective and way of knowing to the analysis of social impact and interdependence. Doing mathematics requires students to "leave their emotions at the door" and to engage in different situations for the purpose of understanding what is really happening and what can be done. Mathematical analysis of topics that interest students such as trends in global warming, homelessness, technological health issues (oil spills, hearing loss, carpal tunnel syndrome), and discrimination can be used to engage the students in interacting and contributing positively to their classroom, school, community, and world. With the understandings that students can derive through mathematical analysis, they become better informed and have a greater respect for and understanding of differing opinions and possible options. With these understandings, students can then make better informed and more personalized decisions regarding roles and contributions to the various communities in which students are members.

Students will contribute to the environmental, economic, and social sustainability of their community through positive life, career, and consumer choices.



Connections to Cross-curricular Competencies

There are four cross-curricular competencies that together contribute to an individual student's development within the four Broad Areas of Learning. It is important therefore, that the learning of mathematics also supports the students in their attainment of these competencies.

Constructing knowledge is how people make sense of the world around them.

Constructing Knowledge

It is important that, within their study of mathematics, students are engaged in personal construction and understandings of mathematical knowledge. This most effectively occurs through student engagement in inquiry and problem solving when challenged to think critically and creatively. Moreover, students need to experience mathematics in a variety of contexts – both real world applications and mathematical contexts – in which students are asked to consider questions such as "what would happen if …", "could we find …", and "what does this tell us?" Students need to be engaged in a social construction of mathematics to develop an understanding and appreciation of mathematics as a tool which can be used to consider different perspectives, connections, and relationships. Mathematics is a subject that depends upon the effective incorporation of independent work and reflection with interactive contemplation, discussion, and resolution.

Identity and Interdependence

Given an appropriate learning environment in mathematics, students can develop both their self-confidence and self-worth. An interactive mathematics classroom in which the ideas, strategies, and abilities of individual students are valued supports the development of personal and mathematical confidence. It can also help students realize their role in the classroom environment and accept responsibility for choices, decisions, and actions. A positive learning environment combined with strong pedagogical choices that engage students in learning serves to support students in behaving respectfully towards themselves and others.

Students need to develop a positive self-concept and to have the ability to live in harmony with others, and with the natural and constructed world.

Developing Literacy

Through their mathematics learning experiences, students should be engaged in developing their understandings of the language of mathematics and their ability to use mathematics as a language. Students should be regularly engaged in exploring a variety of representations for mathematical concepts and should be expected to communicate in a variety of ways about the mathematics being learned. An important part of learning mathematical language is to communicate one's own understandings and to develop strategies to explore what and how others know about mathematics. The study of mathematics should encourage the appropriate use of technology. Moreover, students should be aware of and able to communicate the appropriate use of technology in mathematics and mathematics learning. It is important to encourage students to use a variety of forms of representation (concrete manipulatives, physical movement, visual, and symbolic) when exploring mathematical ideas, solving problems, and communicating understandings. All too often, it is assumed that symbolic representation is the only way to communicate mathematically. The more flexible students are in using a variety of representations to explain and work with the mathematics being learned, the deeper students' understanding becomes.

Mathematics enables individuals to understand and explore their world and communicate and participate in a variety of roles and settings in their home, school, and community.

Social Responsibility

As students progress in their mathematical learning, they need to experience opportunities to share and consider ideas, and resolve conflicts between themselves and others. This requires that the learning environment be co-constructed by the teacher and students to support respectful, independent, and interdependent behaviours. Every student should feel empowered to help others in developing their understanding, while finding respectful ways to seek help from

By encouraging students to explore mathematics in social contexts, students can be engaged in understanding the situation, concern, or issue and then in planning for responsible reactions or responses.

others. By encouraging students to explore mathematics in social contexts, students can be engaged in understanding the situation, concern, or issue and then in planning for responsible reactions or responses. Mathematics is a subject dependent upon social interaction and, as a result, social construction of ideas. Through the study of mathematics, students learn to become reflective and positively contributing members of their communities. Mathematics also allows for different perspectives and approaches to be considered, assessed for situational validity, and strengthened.

Using and interpreting data enables students to make informed, responsible decisions.

Curriculum Integration

There are many possibilities for the integration of mathematics and other subject areas. In doing this integration, however, teachers must be cautious to not lose the integrity of any of the subjects. This integration can be achieved by focussing on the outcomes to be achieved within each area of study. Integration gives students experiences with transfer of knowledge and provides rich contexts in which students are able to make sense of their learnings. Below are just a few of the ways in which mathematics can be integrated into other subject areas (and other subject areas into mathematics) at grade four.

When students experience mathematics as a lens through which to view other subjects, and other subjects as lenses through which to view mathematics, students learn that mathematics is much more than a set of facts and procedures to memorize.

Arts Education – One of the most direct connections between the outcomes for arts education and mathematics at Grade 4 is the focus on symmetry. Both in dance and visual arts, the students are exploring the concept of symmetry (and asymmetry) in movement and design. Dance and visual arts can then be used to help build the students' deep understanding of symmetry as well as becoming additional vehicles through which the students can represent their understandings. The concept of symmetry is also present in drama and music. All four of the Arts Education strands can be used to help students interpret and make sense of their mathematical learnings by having the students explore different ways to represent any of the mathematical ideas being developed. Using the arts to explore confusion and frustration as well as challenges and successes, in mathematics can help to develop student understanding, confidence, and positive attitudes.

English Language Arts – Mathematics and English language arts share the common purpose of providing students with the tools and strategies necessary to communicate ideas in a variety of ways. The use of multiple forms of representation is crucial in both subjects, and students need to learn through practical and hands-on experiences the nuances and conventions of each of these languages. Students should be encouraged to cross the linguistic-representational boundaries between the two disciplines, exploring mathematical texts in English language arts and expressing understandings and questions about mathematics in self-created texts using a variety of forms. Inquiries based upon contextualized questions of a particular form: "what does this tell us?", "how can we say this more effectively or differently?", or "where can we go from here?" can lead students and teachers into an exploration and sense making that creates deep understanding in both subject areas.

Health Education – In health education, the students' exploration of concepts such as caloric intake and serving size based on Canada's Food Guide can be used as real life contexts in which the students can develop their understanding of whole numbers up to 10 000 (including the four operations) and of the meaning and representation of fractions in a variety of contexts. As students analyze their food intake according to Canada's Food Guide or their time devoted to physical exercise, they have a number of opportunities to consider different graphs for the data generated. The students can also look for patterns and relationships in the generated data, as well as making predictions and decisions based upon the data.

Physical Education – There are many possibilities to make connections with mathematics in physical education. The students can generate and record a great deal of data based on measurements related to the health-related components of fitness (about which students are learning). This data can then be analyzed for patterns and relationships, and bar graphs using one-to-one and many-to-one correspondences can be used to represent both individual and class data. The students can then interpret these representations in order to make predictions and generate personal decisions based the data.

Science – In the Habitats and Communities unit in Grade 4 science, students are exploring and making predictions based upon patterns found in the habitats and populations of plants and animals. These patterns can also be analyzed mathematically with students attempting to describe trends mathematically and represent the data graphically. Students can be asked to provide justification through mathematical reasoning for predictions and decisions made based on the situations explored.

Connections can also be made to mathematics in the Sound and Light units by having the students discuss and explore how they can represent relationships and patterns found in investigations. Even if findings are not stated in symbolic mathematical language, the recognition of patterns and relationships in a variety of contexts is foundational to being able to put those patterns and relationships into mathematical terms. In the Rocks, Minerals, and Erosion unit, students can collect first-hand or second-hand data based on different types of rocks and minerals and use this data to create graphical representations with one-to-one and many-to-one correspondences. These graphs can then be used by the students to make predictions and generalizations.

Social Studies – In Grade 4, students are learning about Saskatchewan in Social Studies. Saskatchewan can also be used as a context in which the students explore, develop, and apply their mathematical understandings. Problems related to Saskatchewan places, people, and issues can be used to engage the students in mathematics. Students will also be able to work with their understanding of numbers and data analysis to explore and create a deeper understanding of Saskatchewan within the Social Studies context.

Critical Characteristics of Mathematics Education

The content of K-12 Mathematics can be organized in a variety of ways. In this document, the outcomes and indicators are grouped according to four strands: **Number, Patterns and Relations, Shape and Space, and Statistics and Probability.** Although this organization implies a relatedness between the outcomes identified in each of the strands, it should be noted the mathematical concepts are interrelated between strands as well as within strands.

The mathematics curriculum also recognizes seven processes inherent in the teaching, learning, and doing of mathematics. These processes focus on: communicating, making connections, mental mathematics and estimating, problem solving, reasoning, and visualizing along with using technology to integrate these processes into the mathematics classroom to help students learn mathematics with deeper understanding.

When mathematics is taught without a rich integration of these processes, it becomes a stagnant set of facts and procedures devoid of meaning rather than the dynamic and rich discipline that it is.

The outcomes in K-12 mathematics should be addressed through the appropriate mathematical processes lenses. Teachers should consider carefully in their planning those processes indicated as being important to the various outcomes.

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics.

Students must be able to communicate mathematical ideas in a variety of ways and contexts.

Communication is important in clarifying, reinforcing, and modifying ideas, attitudes, and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology.

Communication can help students make connections among concrete, pictorial, symbolic, verbal, written, and mental representations of mathematical ideas.

Connections [CN]

Through connections to their prior knowledge and daily life, students begin to view mathematics as useful and relevant.

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students begin to view mathematics as useful, relevant, and integrated.

Learning mathematics within contexts and making connections relevant to learners can validate past experiences, and increase student willingness to participate and be actively engaged.

The brain is constantly looking for and making connections. "*Because the learner is constantly searching for connections on many levels, educators need to orchestrate the experiences from which learners extract understanding …. Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching*" (Caine and Caine, 1991, p.5).

Mental mathematics and estimation are fundamental components of number sense.

Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally without the use of external memory aids.

Students proficient with mental mathematics "become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving" (Rubenstein, 2001, p. 442)

Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy, and flexibility.

Estimation is a strategy for determining approximate values of quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when, and what strategy to use when estimating.

Estimation is used to make mathematical judgements and develop useful, efficient strategies for dealing with situations in daily life.

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels.

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, "How would you …?" or "How could you …?", the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not problem solving, but rote practice. A true problem requires students to use prior learnings in new ways and contexts.

Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is a powerful teaching tool that fosters multiple and creative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confidence, reasoning, and mathematical creativity.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and explain their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics.

Mathematical reasoning helps students think logically and make sense of mathematics.

Mathematical experiences in and out of the classroom provide opportunities for inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyze observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Visualization [V]

Visualization "*involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world*" (Armstrong, 1993, p.10). The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them.

Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations.

Visual images and visual reasoning are important components of number, spatial, and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret, and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations.

Technology contributes to the learning of a wide range of mathematical outcomes, and enables students to explore and create patterns, examine relationships, test conjectures and solve problems.

Technology [T]

Technology contributes to the learning of a wide range of mathematical outcomes, and enables students to explore and create patterns, examine relationships, test conjectures, and solve problems.

Calculators and computers can be used to:

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus
- reinforce the learning of basic facts and test properties
- develop personal procedures for mathematical operations
- create geometric displays
- simulate situations
- develop number sense.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. It is important for students to understand and appreciate the appropriate use of technology in a mathematics classroom.

Teaching for Deep Understanding

For deep understanding, it is vital that mathematics be taught through the students uncovering and co-constructing their knowledge, with very few ideas being covered or relayed directly by the teacher. As an example, the notation for fractions ($\frac{a}{b}$) is something which the teacher must cover. It is the sign we use to show that we want to combine or add two quantities. The process of adding, however, and the development of addition and subtraction facts should not be "covered", but rather "discovered" through the students' investigation of patterns, relationships, abstractions, and generalizations. Teachers need to "unpack" outcomes to identify those things that students need to know (behold) and those that they need to be able to apply, explain, and transfer to new situations.

It is very important that a mathematics learning environment include effective interplay of reflection, exploration of patterns and relationships, sharing of ideas and problems, consideration of different perspectives, decision making, generalizing, verifying and proving, and modeling and representing. Mathematics is truly learned when students are engaged in strategic "play" with mathematical concepts and differing perspectives. When students learn mathematics by being told what to do, how to do it, and when to do it, they cannot make the strong learning connections necessary for learning to be meaningful, easily accessible, and transferable.

The mathematics learning environment must necessarily be one which is respectful of individuals and groups. It needs to foster discussion and self-reflection, the asking of questions, the seeking of multiple answers, and the co-construction of meaning.

In grade 4 mathematics, students learn about adding and subtracting up to four digit numbers. Traditionally, teachers prepared students for the stumbling blocks that might be encountered by telling the students about "borrowing" (regrouping) and showing an abstract algorithm in a number of examples. Research shows that, instead, teachers need to approach adding and subtracting up to four-digit whole numbers as a problem for the students to solve. From previous grades, students have a strong understanding of place value and of addition and subtraction as operations; thus, students have the tools to make sense of this new problem. By asking students how to find such sums or differences, the teacher is inviting the student to ***do*** *mathematics which creates their mathematical understanding.*

Grade 4 Mathematics Outcomes and Indicators

Number Strand		
Goals	**Outcomes** *Students will:*	**Indicators** *(The following indicators **may** be used to determine whether students have met the corresponding outcome.)*
Number Sense *Logical Thinking* *Mathematical Attitude*	N4.1 Demonstrate an understanding of whole numbers to 10 000 (pictorially, physically, orally, in writing, and symbolically) by: • representing • describing • comparing two numbers • ordering three or more numbers. [C, R,V]	a) Read a four-digit numeral without using the word “and” (e.g., 5321 is five thousand three hundred twenty one, NOT five thousand three hundred AND twenty one). b) Write a numeral using proper spacing without commas (e.g., 4567 or 4 567, 10 000). c) Write a numeral (0 – 10 000) in words. d) Represent a numeral using a place value chart or diagrams. e) Explain the meaning of each digit in a numeral. f) Express a numeral in expanded notation (e.g., 321 = 300 + 20 + 1). g) Write the numeral represented by an expanded notation expression. h) Explain and show the meaning of each digit in a 4-digit numeral with all digits the same (e.g., for the numeral 2222, the first digit represents two thousands, the second digit two hundreds, the third digit two tens, and the fourth digit two ones). i) Explain the meaning of each digit in a 4-digit number representing a particular quantity. j) Order a set of numbers in ascending or descending order, and explain the order by making references to place value. k) Create and order three different 4-digit numerals. l) Identify the missing numbers in an ordered sequence or shown on a number line. m) Identify incorrectly placed numbers in an ordered sequence or shown on a number line.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[V]**	Visualization
		[T]	Technology

Number Strand		
Goals	**Outcomes** *Students will:*	**Indicators** *(The following indicators **may** be used to determine whether students have met the corresponding outcome.)*
		n) Decompose and represent a 4-digit number at least three different ways. o) Explain why two or more number compositions represent the same quantity.
Number Sense *Logical Thinking* *Mathematical Attitude*	N4.2 Demonstrate an understanding of addition of whole numbers with answers to 10 000 and their corresponding subtractions (limited to 3 and 4-digit numerals) by: • using personal strategies for adding and subtracting • estimating sums and differences • solving problems involving addition and subtraction. [C, CN, ME, PS, R]	a) Explain how to keep track of digits that have the same place value when adding or subtracting numbers. b) Describe a situation in which an estimate rather than an exact answer is sufficient. c) Estimate sums and differences using different strategies (e.g., front-end estimation and compensation). d) Explain the strategies used to determine a sum or difference. e) Solve problems that involve addition and subtraction of more than two numbers.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[V]**	Visualization
		[T]	Technology

Number Strand

Goals	**Outcomes** *Students will:*	**Indicators** *(The following indicators* ***may*** *be used to determine whether students have met the corresponding outcome.)*
Number Sense *Logical Thinking* *Mathematical Attitude*	N4.3 Demonstrate an understanding of multiplication of whole numbers (limited to numbers less than or equal to 10) by: • applying mental mathematics strategies • explaining the results of multiplying by 0 and 1 [C, CN, R]	a) Explain the strategy used to determine a product. b) Explain the strategy used in a given solution to a product. For example: • for 4 x 3, thinking 2 x 3 = 6 and 4 x 3 = 6 + 6 or 6 + 2 = 12 (halving and doubling) • for 3 x 7 think 2 x 7 = 14 and 14 + 7 = 21 (doubling and adding one more group) • for 9 x 6, think 10 x 6 = 60 and 60 – 6 = 54 (multiplying by ten and subtracting one group) • knowing 2 x 6 = 12, then 4 x 6 = 12 x 2 = 24 (doubling) • for 64 ÷ 8, think 8 x □ = 64 (relating division to multiplication) • for 8 x 5, knowing that 5 x 5 = 25, and then skip counting by 5 three times to get 25 + 5 + 5 + 5 = 40. c) Explain the property for determining the answer when multiplying numbers by one. d) Explain the property for determining the answer when multiplying numbers by zero.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[V]**	Visualization
		[T]	Technology

Number Strand		
Goals	**Outcomes** *Students will:*	**Indicators** *(The following indicators* ***may*** *be used to determine whether students have met the corresponding outcome.)*
Number Sense *Logical Thinking* *Mathematical Attitude* *Spatial Sense*	N4.4 Demonstrate an understanding of multiplication (2- or 3-digit by 1-digit) by: • using personal strategies for multiplication, with and without concrete materials • using arrays to represent multiplication • connecting concrete representations to symbolic representations • estimating products • solving problems. [C,ME, PS, R, V]	a) Model a multiplication problem (concretely or symbolically) using the distributive property (e.g., 8 × 365 = (8 × 300) + (8 × 60) + (8 × 5)). b) Use concrete materials, such as base ten blocks or their pictorial representations, to represent multiplication and record the process symbolically. c) Create and solve a multiplication problem that is limited to a 2- or 3-digit number times a 1-digit number. d) Estimate a product using a personal strategy (e.g., 2 × 243 is close to or a little more than 2 × 200, or close to or a little less than 2 × 250). e) Model and solve a multiplication problem using an array, and record the process. f) Solve a multiplication problem and explain the strategies or processes used.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[V]**	Visualization
		[T]	Technology

Number Strand		
Goals	**Outcomes** *Students will:*	**Indicators** *(The following indicators **may** be used to determine whether students have met the corresponding outcome.)*
Number Sense *Logical Thinking* *Mathematical Attitude* *Spatial Sense*	N4.5 Demonstrate an understanding of division (1-digit divisor and up to 2-digit dividend) to solve problems by: • using personal strategies for dividing with and without concrete materials • estimating quotients • explaining the results of dividing by 1 • solving problems involving division of whole numbers • relating division to multiplication. [C, CN, ME, PS, R, V]	*(It is not intended that remainders be expressed as decimals or fractions.)* a) Solve a division problem without a remainder using arrays or base ten materials. b) Solve a division problem with a remainder using arrays or base ten materials. c) Solve a division problem using a personal strategy and record the process symbolically. d) Create and solve a word problem involving a 1- or 2-digit dividend (the number being divided into). e) Estimate a quotient using a personal strategy (e.g., 86 ÷ 4 is close to 80 ÷ 4 or close to 80 ÷ 5). f) Explain the property for determining the answer when dividing numbers by one. g) Explain, using examples, the relationship between division and multiplication.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[V]**	Visualization
		[T]	Technology

Number Strand		
Goals	**Outcomes** *Students will:*	**Indicators** *(The following indicators **may** be used to determine whether students have met the corresponding outcome.)*
Number Sense *Logical Thinking* *Mathematical Attitude* *Spatial Sense*	N4.6 Demonstrate an understanding of fractions less than or equal to one by using concrete and pictorial representations to: • name and record fractions for the parts of a whole or a set • compare and order fractions • model and explain that for different wholes, two identical fractions may not represent the same quantity • provide examples of where fractions are used. [C, CN, PS, R, V]	a) Represent a fraction using concrete materials. b) Represent a fraction based on a symbolically concrete representation (e.g., circles for cookies). c) Name and record the fraction for the included and not included parts of a set. d) Name and record the shaded and non-shaded (included and not included) parts of a whole. e) Represent a fraction pictorially by indicating parts of a given set. f) Represent a fraction pictorially by indicating parts of a whole. g) Explain how denominators can be used to compare two unit fractions with numerator 1. h) Order a set of fractions that have the same numerator and explain the ordering. i) Order a set of fractions that have the same denominator and explain the ordering. j) Identify which of the benchmarks $0, \frac{1}{2}, \text{or } 1$ is closer to a given fraction. k) Name fractions between two benchmarks on a number line. l) Order a set of fractions by placing them on a number line with given benchmarks. m) Provide examples of when two identical fractions may not represent the same quantity (e.g., half of a large apple is not equivalent to half of a small apple; half a group of ten cloudberries is not equivalent to half of a group of sixteen cloudberries). n) Provide an example of a fraction that represents part of a set, a fraction that represents part of a whole, or a fraction that represents part of a length from everyday contexts.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[V]**	Visualization
		[T]	Technology

Number Strand

Goals	**Outcomes** *Students will:*	**Indicators** *(The following indicators **may** be used to determine whether students have met the corresponding outcome.)*
Number Sense *Logical Thinking* *Mathematical Attitude* *Spatial Sense*	N4.7 Demonstrate an understanding of decimal numbers in tenths and hundredths (pictorially, orally, in writing, and symbolically) by: • describing • representing • relating to fractions. [C, CN, V]	a) Write the decimal for a concrete or pictorial representation of part of a set, part of a region, or part of a unit of measure. b) Represent a decimal concretely or pictorially. c) Explain the meaning of each digit in a given decimal with all digits the same. d) Represent a decimal using money (dimes and pennies). e) Record a money value using decimals. f) Provide examples of everyday contexts in which tenths and hundredths are used. g) Model, using manipulatives or pictures, that a tenth can be expressed as hundredths (e.g., 0.9 is equivalent to 0.90 or 9 dimes is equivalent to 90 pennies). h) Read and write decimals as fractions (e.g., 0.5 is zero and five tenths). i) Express orally and in symbolic form a decimal in fractional form. j) Express orally and in symbolic form a fraction with a denominator of 10 or 100 as a decimal. k) Express a pictorial or concrete representation as a fraction or decimal (e.g., 15 shaded squares on a hundred grid can be expressed as 0.15 or $\frac{15}{100}$). l) Express orally and in symbolic form the decimal equivalent for a fraction (e.g., $\frac{50}{100}$ can be expressed as 0.50).

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[V]**	Visualization
		[T]	Technology

Number Strand		
Goals	**Outcomes** *Students will:*	**Indicators** *(The following indicators* ***may*** *be used to determine whether students have met the corresponding outcome.)*
Number Sense *Logical Thinking* *Mathematical Attitude* *Spatial Sense*	N4.8 Demonstrate an understanding of addition and subtraction of decimals limited to hundredths (concretely, pictorially, and symbolically) by: • using compatible numbers • estimating sums and differences • using mental math strategies • solving problems. [C, ME, PS, R, V]	a) Approximate sums and differences of decimals using estimation strategies. b) Solve problems, including money problems, which involve addition and subtraction of decimals, limited to hundredths. c) Determine the approximate solution of a problem not requiring an exact answer. d) Estimate a sum or difference using compatible numbers. e) Count back change for a purchase. f) Explain the strategies used to determine a sum or difference. g) Represent a sum or difference of two decimals concretely or pictorially, and record the solution to the sum or difference symbolically.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[V]**	Visualization
		[T]	Technology

Patterns and Relations Strand		
Goals	**Outcomes** *Students will:*	**Indicators** *(The following indicators **may** be used to determine whether students have met the corresponding outcome.)*
Number Sense *Logical Thinking* *Mathematical Attitude* *Spatial Sense*	P4.1 Demonstrate an understanding of patterns and relations by: • identifying and describing patterns and relations in a chart, table or diagram • reproducing patterns and relations in a chart, table, or diagram using manipulatives • creating charts, tables, or diagrams to represent patterns and relations • solving problems involving patterns and relations [C, CN, PS, R]	a) Identify and describe a variety of patterns in a multiplication chart. b) Determine the missing element(s) in a table or chart and explain the strategies used. c) Identify and correct error(s) in a table or chart. d) Describe the pattern found in a table or chart. e) Create a concrete representation of a pattern displayed in a table or chart. f) Explain why the same relationships exist within a pattern in a table and its concrete representation. g) Extend patterns found in a table or chart to solve a problem. h) Translate the information provided in a problem into a table or chart. i) Identify and extend the patterns in a table or chart to solve a problem. j) Solve a problem by completing a Carroll diagram using given data. k) Determine where new data belong in a Carroll diagram. l) Identify the sorting rule for a Venn diagram. m) Describe the relationship shown in a given Venn diagram when the circles intersect, when one circle is contained in the other, and when the circles are separate. n) Determine where new data belong in a Venn diagram. o) Solve a problem by using a chart or diagram to identify mathematical relationships.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[V]**	Visualization
		[T]	Technology

Patterns and Relations Strand		
Goals	**Outcomes** *Students will:*	**Indicators** *(The following indicators **may** be used to determine whether students have met the corresponding outcome.)*
Number Sense *Logical Thinking* *Spatial Sense* *Mathematical Attitude*	P4.2 Demonstrate an understanding of equations involving symbols to represent an unknown value by: • writing an equation to represent a problem • solving one step equations. [C, ME, PS, R]	a) Explain the purpose of the symbol, such as a triangle or circle, in an addition, subtraction, multiplication, or division equation with one unknown (e.g. 36 ÷ □ = 6). b) Write an equation in symbolic form for a given pictorial or concrete representation. c) Identify the unknown in a story problem, represent the problem with an equation, and solve the problem concretely, pictorially, or symbolically. d) Create a problem in context for an equation with one unknown. e) Solve a one-step equation using manipulatives. f) Solve a one-step equation using guess and test. g) Explain what is meant by "one-step equation with one unknown". h) Represent and solve an addition or subtraction problem involving a "part-part-whole" or comparison context using a symbol to represent the unknown. i) Represent and solve a multiplication or division problem involving equal grouping or partitioning (equal sharing) using a symbol to represent the unknown.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[V]**	Visualization
		[T]	Technology

Shape and Space Strand		
Goals	**Outcomes** *Students will:*	**Indicators** *(The following indicators **may** be used to determine whether students have met the corresponding outcome.)*
Number Sense *Logical Thinking* *Mathematical Attitude*	SS4.1 Demonstrate an understanding of time by: • reading and recording time using digital and analog clocks (including 24 hour clocks) • reading and recording calendar dates in a variety of formats. [C, CN, V]	a) State the number of hours in a day. b) Express the time orally and numerically shown on a 12-hour analog clock. c) Express the time orally and numerically shown on a 24-hour analog clock. d) Express the time orally shown on a 12-hour digital clock. e) Express time orally shown on a 24-hour digital clock. f) Express time orally as “minutes to” or “minutes after” the hour. g) Explain the meaning of AM and PM, and provide an example of an activity that occurs during the AM and another that occurs during the PM. h) Write dates in a variety of formats (e.g., yyyy/mm/dd; dd/mm/yyyy; March 21, 2006; dd/mm/yy). i) Relate dates written in the format yyyy/mm/dd to dates on a calendar. j) Identify possible interpretations of a date (e.g., 06/03/04).

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[V]**	Visualization
		[T]	Technology

Shape and Space Strand		
Goals	**Outcomes** *Students will:*	**Indicators** *(The following indicators **may** be used to determine whether students have met the corresponding outcome.)*
Spatial Sense *Number Sense* *Logical Thinking* *Mathematical Attitude*	SS4.2 Demonstrate an understanding of area of regular and irregular 2-D shapes by: • recognizing that area is measured in square units • selecting and justifying referents for the units cm^2 or m^2 • estimating area by using referents for cm^2 or m^2 • determining and recording area (cm^2 or m^2) • constructing different rectangles for a given area (cm^2 or m^2) in order to demonstrate that many different rectangles may have the same area. [C, CN, ME, PS, R, V]	a) Describe area as the measure of surface recorded in square units. b) Identify and explain why the square is a most efficient unit for measuring area. c) Provide a referent for a square centimetre and explain the choice. d) Provide a referent for a square metre and explain the choice. e) Determine which standard square unit is represented by a referent. f) Estimate the area of a 2-D shape using personal referents. g) Determine the area of a regular 2-D shape and explain the strategy used. h) Determine the area of an irregular 2-D shape and explain the strategy used. i) Construct a rectangle with a given area. j) Illustrate, and verify, how more than one rectangle is possible for a given area by drawing at least two different rectangles with that area (e.g., identifying the dimensions of each rectangle drawn, or superimpose the rectangles on each other).

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[V]**	Visualization
		[T]	Technology

Shape and Space Strand		
Goals	**Outcomes** *Students will:*	**Indicators** *(The following indicators **may** be used to determine whether students have met the corresponding outcome.)*
Spatial Sense *Logical Thinking* *Mathematical Attitude*	SS4.3 Demonstrate an understanding of rectangular and triangular prisms by: • identifying common attributes • comparing • constructing models. [C, CN, R, V]	a) Identify and name common attributes of rectangular prisms from sets of rectangular prisms. b) Identify and name common attributes of triangular prisms from sets of triangular prisms. c) Sort a set of rectangular and triangular prisms using the shape of the base. d) Identify examples of rectangular and triangular prisms found in the environment. e) Construct and describe a model of rectangular and triangular prisms. f) Construct rectangular prisms from their nets. g) Construct triangular prisms from their nets. h) Construct nets for rectangular or triangular prisms.
Spatial Sense *Logical Thinking* *Mathematical Attitude*	SS4.4 Demonstrate an understanding of line symmetry by: • identifying symmetrical 2-D shapes • creating symmetrical 2-D shapes • drawing one or more lines of symmetry in a 2-D shape. [C, CN, V]	a) Identify the characteristics of given symmetrical and non-symmetrical 2-D shapes. b) Sort a set of 2-D shapes as symmetrical and non-symmetrical. c) Complete a symmetrical 2-D shape given half the shape and its line of symmetry. d) Explain how symmetry and fractions are related. e) Identify lines of symmetry in a set of 2-D shapes and explain why each shape is symmetrical. f) Determine whether or not a given 2-D shape is symmetrical by using a Mira or by folding and superimposing. g) Create a symmetrical shape with and without manipulatives. h) Provide examples of symmetrical shapes found in the environment and identify the line(s) of symmetry.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[V]**	Visualization
		[T]	Technology

Statistics and Probability Strand		
Goals	**Outcomes** *Students will:*	**Indicators** *(The following indicators **may** be used to determine whether students have met the corresponding outcome.)*
		i) Sort a given set of 2-D shapes as those that have no lines of symmetry, one line of symmetry, or more than one line of symmetry.
Spatial Sense *Number Sense* *Logical Thinking* *Mathematical Attitude*	SP4.1 Demonstrate an understanding of many-to-one correspondence by: • comparing correspondences on graphs • justifying the use of many-to-one correspondences • interpreting data shown using a many-to-one correspondence • creating bar graphs and pictographs using many-to-one correspondence. [C, R, T, V]	a) Compare graphs in which different correspondences are used and explain why the correspondence may have been used. b) Compare graphs in which the same data have been displayed using a one-to-one and a many-to-one correspondence, and explain how they are the same and different. c) Explain why a many-to-one correspondence is sometimes used rather than a one-to-one correspondence. d) Find examples of graphs in which a many-to-one correspondence is used in print and electronic media, such as newspapers, magazines, and the Internet, and describe the correspondence used. e) Select many-to-one correspondence for displaying a set of data in a graph and justify the choice. f) Create and label (with categories, title, and legend) a pictograph to display a set of data using a many-to-one correspondence, and justify the choice of correspondence used. g) Create and label (with axes and title) a bar graph to display a set of data using a many-to-one correspondence, and justify the choice of correspondence used. h) Answer a question using a graph in which data are displayed using a many-to-one correspondence.

Appendix A: Terminology

Attributes: Characteristics of 2-D shapes and 3-D objects that can be used to compare and sort sets of 2-D shapes and 3-D objects (e.g., colour, relative size, number of corners, number of lines of symmetry, etc.).

Benchmarks: Numeric quantities used to compare and order other numeric quantities. For example, 0, ½, and 1 are often used as benchmarks when placing fractions between 0 and 1 on a number line.

Carroll Diagram: A table used for organizing and highlighting relationships between characteristics of elements in a data set. Each characteristic is broken into a yes/no descriptors or into independent categories. The Carroll diagram shows all possible ways to match the different categories between the characteristics. For example, the following Carroll diagram represents information about a classroom in terms of girls and boys with sandals or shoes:

	Boy	Girl
Shoes	7	3
Sandals	2	4

These diagrams can be used to analyze a situation according to one characteristic, one category, or a combination of both.

Chart: "Chart" is a general word for various kinds of pictures and diagrams which are used to represent data – often frequency data (e.g., a bar graph can be referred to as a bar chart).

Compatible Numbers: When performing a calculation students are encouraged to look for compatible numbers – numbers with which it is easy to work (e.g., when adding 375, 215, and 125, students might see the 75 in 375 and the 25 in 125 as compatible because they add to 100). They would use this information to continue to find the sum using a variety of mental mathematics strategies.

Compensation: Making adjustments to solutions or answers based on adjustments made to the starting values. For example, if in adding 148 and 27 the sum 150 +30 = 180 was determined, as 150 is 2 more than 148 and 30 is 3 more than 27, the sum of 180 must be compensated by subtracting 2 and 3, 180 – 2 – 3 = 180 –5 = 175.

Correspondence (many-to-one and one-to-one): A correspondence is a description of how one set of numbers (or objects) is mapped to a second set of objects. For example, a correspondence might describe how individual students are matched to their shoes. If each child in the class has a different type of shoe, then the correspondence between the shoes and the students would be one-to-one (for every child, there is exactly one type of shoe). If some children have the same type of shoe, then the correspondence is said to be many-to-one (many children to one type of shoe).

Distributive Property (of multiplication): Multiplication is said to be distributive over addition and subtraction because a sum or difference times a number is the same as the sum or difference of the products (e.g., 3 x (5 – 2) = (3 x 5) – (3 x 2)). In multiplying a 2-digit number by a one digit number, the distributive property of multiplication over addition can be applied (e.g., 8 x 34 = 8 x (30+4) = (8 x 30) + (8 x 4)).

Equation (one step, one unknown): A statement of equality that has only one symbol in it (not a variable or letter) and the solution of the equation requires only one step of either addition, subtraction, multiplication, or division.

Front-end Estimation: A process of finding approximate values for computations by considering rounded or sometimes truncated values (e.g., 34 + 72 is approximately equal to 30 + 70 = 100 by front-end estimation).

Line Symmetry: A property of some 2-D shapes in which a line can be used to divide the 2-D shape in half. Although there are many types of symmetry, in grade four the students are focussing on symmetry that can be defined as a reflection through the line of symmetry (i.e., no rotation or translation involved).

Number, Numeral, Digit: A number is the name that we give to quantities (e.g., there are 7 days in a week, or I have three brothers – both seven and three are numbers in these situations because they are defining a quantity). The symbolic representation of a number, such as 287, is called the numeral. If 287 is not being used to define a quantity, we call it a numeral.

Numerals, as the symbolic representation of numbers, are made up of a series of digits. The Hindu-Arabic number system that we use has ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. (Note: sometimes students are confused between these digits and their finger digits – this is because they count their fingers starting at one and get to ten rather than zero to nine). These digits are also numerals and can be numbers (representing a quantity), but all numbers and all numerals are combinations of digits. The placement of a digit in a number or numeral affects the place value of the digit, and hence how much of the quantity that it represents. For example, in 326, the 2 is contributing 20 to the total while in 236, the 2 contributes 200 to the total quantity.

Number Line: Because grade four students have not been introduced to scale or ratio and proportion, number lines should reflect relative positioning rather than scaled points. In some resources, the use of a number line that indicates the relationship between numbers (but not the ratio of quantity) is called an *Empty Number Line.*

Part-part-whole Context: A problem situation that describes a whole in relation to two parts.

Personal Strategies: Personal strategies are strategies that the students have constructed and understand. Outcomes and indicators that specify the use of personal strategies convey the message that there is not a single procedure that is correct. Students should be encouraged to explore, share, and make decisions about what strategies to use in different contexts. Development of personal strategies is an indicator of the attainment of a deeper understanding.

Prisms (rectangular and triangular): Prisms are 3-D objects that have two bases opposite each other on the 3-D object that are the same size and shape. The prisms are named according to the shape of the base (e.g., a triangular prism is a prism that has a triangular base). A water trough is a common example of a triangular prism. All prisms that grade 4 students are studying are right prisms – that is their sides are perpendicular to the base (i.e., the object does not lean).

Referents: A concrete representation of a unit of measure. For example, a dime might be used as a referent for 1 cm^2. Referents are used to determine estimates of measurements.

Regular/irregular 2-D Shapes: Regular 2-D shapes are those shapes whose side lengths are equal and angle measures are also equal. Irregular 2-D shapes do not have all equal side lengths and/or all equal angle measures. For example, a square is a regular 2-D shape, while not all rectangles are regular 2-D shapes.

Relation: A statement that explains how the terms in a pattern relate to each other or how one unknown value is related to another.

Representation: Mathematical ideas can be represented and manipulated in a variety of formats including concrete manipulatives, visual design, physical movements, and symbolic notation. Students need to have experiences in working with many different types of representations and in transferring and translating knowledge between the different forms of representations.

Superimposing: This term refers to the covering of one 2-D shape by one or more other 2-D shapes by placing the new shapes on top of the original. In mathematics, this can be done to determine fractional relationships, equivalent areas, compare areas, and symmetry.

Symbols to Represent an Unknown Value: In grade four, students should be using symbols (such as boxes or circles) to represent unknown values. Letters, such as x or y, should *not* be used in grade 4 because the students are in the process of constructing their understanding of an unknown and the introduction of letters (or variables) into equations can cause students frustrations and misunderstandings.

Table: A set of data specially laid out in rows and column, so that given one value, another separate value can be read out which is connected to the first one in some way (e.g., multiplication table).

Appendix B: Three Grades at a Glance

The chart below show the outcomes of mathematics in grades 3, 4, and 5 in each of the four strands with the outcomes lined up to show the flow of content development.

<table>
<tr><th colspan="3">Number Strand</th></tr>
<tr><th>Grade 3 (Draft)</th><th>Grade 4</th><th>Grade 5 (Draft)</th></tr>
<tr><th colspan="3">Whole Numbers</th></tr>
<tr><td>Represent and describe numbers to 1000, orally, concretely, visually, physically, and symbolically.
[C, CN, V]

Compare (two numbers) and order (three or more numbers) numbers to 1000.
[CN, R, V]

Estimate quantities less than 1000 using referents (known quantities).
[ME, PS, R, V]

Illustrate, concretely and pictorially, the meaning of place value for numerals to 1000.
[C, CN, R, V]</td><td>N4.1 Demonstrate an understanding of whole numbers to 10 000 (pictorially, physically, orally, in writing, and symbolically) by:
• representing
• describing
• comparing two numbers
• ordering three or more numbers.
[C, R,V]</td><td>Represent and describe whole numbers to 1 000 000.
[C, CN, V, T]</td></tr>
</table>

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[V]**	Visualization
		[T]	Technology

Number Strand		
Grade 3 *(Draft)*	**Grade 4**	**Grade 5 *(Draft)***
Adding and Subtracting Whole Numbers		
Describe and apply mental mathematics strategies for adding two 2-digit numerals, such as: • adding from left to right • taking one addend to the nearest multiple of ten and then compensating • using doubles. [C, ME, PS, R, V] Describe and apply mental mathematics strategies for subtracting two 2-digit numerals, such as: • taking the subtrahend to the nearest multiple of ten and then compensating • thinking of addition • using doubles. [C, ME, PS, R, V] Apply estimation strategies to predict sums and differences of two 2-digit numerals in a problem-solving context. [C, ME, PS, R]	N4.2 Demonstrate an understanding of addition of numbers with answers to 10 000 and their corresponding subtractions (limited to 3 and 4-digit numerals) by: • using personal strategies for adding and subtracting • estimating sums and differences • solving problems involving addition and subtraction. [C, CN, ME, PS, R]	

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[V]**	Visualization
		[T]	Technology

Number Strand		
Grade 3 *(Draft)*	**Grade 4**	**Grade 5 *(Draft)***
Adding and Subtracting Whole Numbers		
Demonstrate an understanding of addition and subtraction of numbers with answers to 1000 (limited to 1, 2, and 3-digit numerals) by: • using personal strategies for adding and subtracting with and without the support of manipulatives • creating and solving problems in contexts that involve addition and subtraction of numbers • concretely, pictorially, and symbolically. [C, CN, ME, PS, R] Apply mental mathematics strategies and number properties, such as: • using doubles • making 10 • using the commutative property • using the property of zero • thinking addition for subtraction to recall basic addition facts to 18 and related subtraction facts. [C, CN, ME, R, V]		

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[V]**	Visualization
		[T]	Technology

Number Strand		
Grade 3 *(Draft)*	**Grade 4**	**Grade 5 *(Draft)***
Multiplication and Division of Whole Numbers		
Demonstrate an understanding of multiplication to 5 × 5 by: • representing and explaining multiplication using equal grouping and arrays • creating and solving problems in context that involve multiplication • modelling multiplication using concrete and visual representations, and recording the process symbolically • relating multiplication to repeated addition • relating multiplication to division. [C, CN, PS, R]	N4.3 Demonstrate an understanding of multiplication of whole numbers (limited to numbers less than or equal to 10) by: • applying mental mathematics strategies • explaining the results of multiplying by 0 and 1 • explaining the results of dividing by 1. [C, CN, R] N4.4. Demonstrate an understanding of multiplication (2- or 3-digit by 1-digit) by: • using personal strategies for multiplication with and without concrete materials • using arrays to represent multiplication • connecting concrete representations to symbolic representations • estimating products • solving problems. [C,ME, PS, R, V]	Apply mental mathematics strategies and number properties, such as: • skip counting from a known fact • using doubling or halving • using patterns in the 9s facts • using repeated doubling or halving to determine answers for basic multiplication facts to 81 and related division facts. [C, CN, ME, R, V] Apply mental mathematics strategies for multiplication, such as: • annexing then adding zero • halving and doubling • using the distributive property. [C, ME, R] Demonstrate an understanding of multiplication (2-digit by 2-digit) to solve problems. [C, CN, PS, V]

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[V]**	Visualization
		[T]	Technology

Number Strand		
Grade 3 *(Draft)*	**Grade 4**	**Grade 5 *(Draft)***
Whole Numbers		
Demonstrate an understanding of division by: • representing and explaining division using equal sharing and equal grouping • creating and solving problems in context that involve equal sharing and equal grouping • modelling equal sharing and equal grouping using concrete and visual representations, and recording the process symbolically • relating division to repeated subtraction • relating division to multiplication (limited to division related to multiplication facts up to 5 × 5) [C, CN, PS, R]	N4.5 Demonstrate an understanding of division (1-digit divisor and up to 2-digit dividend) to solve problems by: • using personal strategies for dividing, with and without concrete materials • estimating quotients • explaining the results of dividing by 1 • solving problems involving division of whole numbers • relating division to multiplication. [C, CN, ME, PS, R, V]	Demonstrate, with and without concrete materials, an understanding of division (3-digit by 1-digit) and interpret remainders to solve problems. [C, CN, PS]
Estimation Strategies for Operations on Whole Numbers		
		Use estimation strategies including: • front-end rounding • compensation • compatible numbers in problem-solving contexts. [C, CN, ME, PS, R, V]

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[V]**	Visualization
		[T]	Technology

Number Strand		
Grade 3 *(Draft)*	**Grade 4**	**Grade 5 *(Draft)***
Fractions		
Demonstrate an understanding of fractions by: • explaining that a fraction represents a part of a whole • describing situations in which fractions are used • comparing fractions of the same whole with like denominators. [C, CN, ME, R, V]	N4.6 Demonstrate an understanding of fractions less than or equal to one by using concrete and pictorial representations to: • name and record fractions for the parts of a whole or a set • compare and order fractions • model and explain that for different wholes, two identical fractions may not represent the same quantity • provide examples of where fractions are used. [C, CN, PS, R, V]	Demonstrate an understanding of fractions by using concrete and pictorial representations to: • create sets of equivalent fractions • compare fractions with like and unlike denominators. [C, CN, PS, R, V]
Decimals and Fractions		
	N4.7 Demonstrate an understanding of decimal numbers in tenths and hundredths (pictorially, orally, in writing, and symbolically) by: • describing • representing • relating to fractions. [C, CN, V]	Relate decimals to fractions (to thousandths). [CN, R, V] Compare and order decimals (to thousandths) by using: • benchmarks • place value • equivalent decimals. [CN, R, V]

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[V]**	Visualization
		[T]	Technology

Number Strand		
Grade 3 *(Draft)*	**Grade 4**	**Grade 5 *(Draft)***
Addition and Subtraction of Decimals		
	N4.8 Demonstrate an understanding of addition and subtraction of decimals limited to hundredths (concretely, pictorially and symbolically) by: • using compatible numbers • estimating sums and differences • using mental math strategies • solving problems. [C, ME, PS, R, V]	Demonstrate an understanding of addition and subtraction of decimals (limited to thousandths). [C, CN, PS, R, V]

Patterns and Relations Strand		
Grade 3 *(Draft)*	**Grade 4**	**Grade 5 *(Draft)***
Increasing and Decreasing Patterns		
Demonstrate an understanding of increasing patterns by: • describing • extending • comparing • creating patterns using manipulatives, diagrams, sounds, and actions (numbers to 1000). [C, CN, PS, R, V] Demonstrate an understanding of decreasing patterns by: • describing • extending • comparing • creating patterns using manipulatives, diagrams, sounds, and actions (numbers to 1000). [C, CN, PS, R, V]		

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[V]**	Visualization
		[T]	Technology

Patterns and Relations Strand		
Grade 3 *(Draft)*	**Grade 4**	**Grade 5 *(Draft)***
Patterns and Pattern Rules in Tables and Charts		
	P4.1 Demonstrate an understanding of patterns and relations by: • identifying and describing patterns and relations in a chart, table, or diagram • reproducing patterns and relations in a chart, table, or diagram using manipulatives • creating charts, tables, or diagrams to represent patterns and relations • solving problems involving patterns and relations [C, CN, PS, R]	Determine the pattern rule to make predictions about subsequent elements. [C, CN, PS, R, V]
Solving Equations involving Symbols		
Solve one-step addition and subtraction equations involving symbols representing an unknown number. [C, CN, PS, R, V]	P4.2 Demonstrate an understanding of equations involving symbols to represent an unknown value by: • writing an equation to represent a problem • solving one step equations. [C, ME, PS, R]	
Solving Equations involving Variables		
		Solve problems involving single-variable, one-step equations with whole number coefficients and whole number solutions. [C, CN, PS, R]

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[V]**	Visualization
		[T]	Technology

Shape and Space Strand		
Grade 3 *(Draft)*	**Grade 4**	**Grade 5 *(Draft)***
Time		
Relate the passage of time to common activities using non-standard and standard units (minutes, hours, days, weeks, months, years). [CN, ME, R] Relate the number of seconds to a minute, the number of minutes to an hour, and the number of days to a month in a problem-solving context. [C, CN, PS, R, V]	SS4.1 Demonstrate an understanding of time by: • reading and recording time, using digital and analog clocks (including 24 hour clocks) • reading and recording calendar dates in a variety of formats. [C, CN, V]	
Mass		
Demonstrate an understanding of measuring mass (g, kg) by: • selecting and justifying referents for the units g and kg • modelling and describing the relationship between the units g and kg • estimating mass using referents • measuring and recording mass. [C, CN, ME, PS, R, V]		

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[V]**	Visualization
		[T]	Technology

Shape and Space Strand		
Grade 3 *(Draft)*	**Grade 4**	**Grade 5 *(Draft)***
Length and Perimeter		
Demonstrate an understanding of measuring length (cm, m) by: • selecting and justifying referents for the units cm and m • modelling and describing the relationship between the units cm and m • estimating length using referents • measuring and recording length, width, and height. [C, CN, ME, PS, R, V] Demonstrate an understanding of perimeter of regular and irregular shapes by: • estimating perimeter using referents for centimetre or metre • measuring and recording perimeter (cm, m) • constructing different shapes for a given perimeter (cm, m) to demonstrate that many shapes are possible for a perimeter. [C, ME, PS, R, V]		Demonstrate an understanding of measuring length (mm) by: selecting and justifying referents for the unit mm • modelling and describing the relationship between mm and cm units, and between mm and m units. [C, CN, ME, PS, R, V]

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[V]**	Visualization
		[T]	Technology

Shape and Space Strand		
Grade 3 *(Draft)*	**Grade 4**	**Grade 5 *(Draft)***
	Area	
	SS4.2 Demonstrate an understanding of area of regular and irregular 2-D shapes by: • recognizing that area is measured in square units • selecting and justifying referents for the units cm^2 or m^2 • estimating area by using referents for cm^2 or m^2 • determining and recording area (cm^2 or m^2) • constructing different rectangles for a given area (cm^2 or m^2) in order to demonstrate that many different rectangles may have the same area. [C, CN, ME, PS, R, V]	
	Perimeter and Area	
		Design and construct different rectangles given either perimeter or area, or both (whole numbers) and draw conclusions. [C, CN, PS, R, V]

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[V]**	Visualization
		[T]	Technology

Shape and Space Strand		
Grade 3 *(Draft)*	**Grade 4**	**Grade 5 *(Draft)***
Volume		
		Demonstrate an understanding of volume by: • selecting and justifying referents for cm^3 or m^3 units • estimating volume by using referents for cm^3 or m^3 • measuring and recording volume (cm^3 or m^3) • constructing rectangular prisms for a given volume. [C, CN, ME, PS, R, V]
Capacity		
		Demonstrate an understanding of capacity by: • describing the relationship between mL and L • selecting and justifying referents for mL or L units • estimating capacity by using referents for mL or L • measuring and recording capacity (mL or L). [C, CN, ME, PS, R, V]
3-D Objects		
Describe 3-D objects according to the shape of the faces, and the number of edges and vertices. [C, CN, PS, R, V]	SS4.3 Demonstrate an understanding of rectangular and triangular prisms by: • identifying common attributes • comparing • constructing models. [C, CN, R, V]	

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[V]**	Visualization
		[T]	Technology

<table>
<tr><th colspan="3">Shape and Space Strand</th></tr>
<tr><th>Grade 3 (Draft)</th><th>Grade 4</th><th>Grade 5 (Draft)</th></tr>
<tr><th colspan="3">2-D Shapes</th></tr>
<tr><td>Sort regular and irregular polygons, including:
• triangles
• quadrilaterals
• pentagons
• hexagons
• octagons
according to the number of sides.
[C, CN, R, V]</td><td></td><td>Identify and sort quadrilaterals, including:
• rectangles
• squares
• trapezoids
• parallelograms
• rhombuses
according to their attributes.
[C, R, V]</td></tr>
<tr><th colspan="3">Symmetry</th></tr>
<tr><td></td><td>SS4.4 Demonstrate an understanding of line symmetry by:
• identifying symmetrical 2-D shapes
• creating symmetrical 2-D shapes
• drawing one or more lines of symmetry in a 2-D shape.
[C, CN, V]</td><td></td></tr>
<tr><th colspan="3">3-D Objects and 2-D Shapes</th></tr>
<tr><td></td><td></td><td>Describe and provide examples of edges and faces of 3-D objects, and sides of 2-D shapes that are:
• parallel
• intersecting
• perpendicular
• vertical
• horizontal.
[C, CN, R, T, V]</td></tr>
</table>

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[V]**	Visualization
		[T]	Technology

Shape and Space Strand		
Grade 3 *(Draft)*	**Grade 4**	**Grade 5 *(Draft)***
Transformations		
		Perform a single transformation (translation, rotation, or reflection) of a 2-D shape (with and without technology) and draw and describe the image. [C, CN, T, V] Identify a single transformation, including a translation, rotation, and reflection of 2-D shapes. [C, T, V]

Statistics and Probability Strand		
Grade 3 *(Draft)*	**Grade 4**	**Grade 5 *(Draft)***
Data collections and types		
Collect first-hand data and organize it using: • tally marks • line plots • charts • lists to answer questions. [C, CN, V]		Differentiate between first-hand and second-hand data. [C, R, T, V]

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[V]**	Visualization
		[T]	Technology

Statistics and Probability Strand		
Grade 3 *(Draft)*	**Grade 4**	**Grade 5 *(Draft)***
Data Displays and Interpretation		
Construct, label, and interpret bar graphs to solve problems. [PS, R, V]	SP4.1 Demonstrate an understanding of many-to-one correspondence • comparing correspondences on graphs • justifying the use of many-to-one correspondences • interpreting data shown using a many-to-one correspondence • creating data graphs (bar and pictographs) using many-to-one correspondence. [C, R, T, V]	Construct and interpret double bar graphs to draw conclusions. [C, PS, R, T, V]
Probability of Single Events		
		Describe the likelihood of a single outcome occurring using words, such as: • impossible • possible • certain. [C, CN, PS, R] Compare the likelihood of two possible outcomes occurring using words, such as: • less likely • equally likely • more likely. [C, CN, PS, R]

References

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